Simultaneous identification of noise and estimation of noise standard deviation in MRI

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Outline

Introduction

- Data analysis in MRI is a 'pipeline' of closely connected processing and modeling steps.
- Because noise in MRI data affects all subsequent steps in this pipeline, accurate noise assessment is important in MRI studies.
- Noise assessment in MRI usually means the estimation of noise variance (or standard deviation (SD)) alone.
- In this work, we propose a simple approach, called PIESNO, to simultaneously identify noise-only pixels and estimate the noise standard deviation from these pixels.

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Noise Assessment in MRI

- Noise assessment in MRI usually means the estimation of Gaussian noise variance (or standard deviation (SD)) alone (1-7).
- Previously proposed methods on noise assessment can be separated into two groups:
  1. Region-of-interest (ROI) based methods through manual selection of ROI.
  2. Automatic (without human intervention) methods that use an entire image or volumetric data set for noise estimation.

Pros and Cons of the ROI-based Methods

These methods date back to the 80's and 90's, e.g.,(1,2,3,7).

Pros

- The methods are generally easy to use.
- They are computationally very efficient because they are based on the method of moments.
- They can be used on small sample.

Cons

- They require human invention (to draw an ROI).
- The results are not easily reproducible.

Pros and Cons of Current Automatic Methods

The group of methods were studied by Profs. Chang (4) and Sijbers (5).

Pros

- The methods are automatic—entire image rather than ROI is used in the estimation.
- The results are reproducible.

Cons

- They usually require sufficiently large sample because they are histogram-based methods.
- The separation of pure noise from noisy signals and other artifacts is poor in certain cases.

Outline

Observations

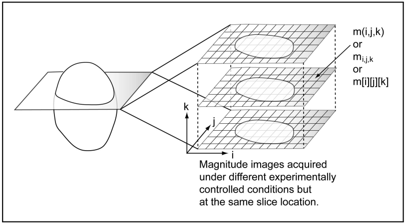
- The most critical problem facing current automatic methods is the separation of pure noise from noisy signals and other artifacts.
- Current automatic methods work by lumping the values of all the pixels from an entire image or from an entire volumetric data set into a one-dimensional array and then estimating the Gaussian noise SD from the histogram of this one-dimensional array.
- Complicated criteria and techniques have been developed to separate pure noise from noisy signals and other artifacts from the histogram alone.

Outline

Ideas for a New Paradigm in Noise Assessment

- The identification of noise-only pixels should be a part of the paradigm in order to enhance the performance and accuracy of the estimation process.
- The identification of noise-only pixels entails some a priori knowledge of the Gaussian noise SD.
- Therefore, any paradigm that attempts to make the identification of noise-only pixels a part of the overall noise assessment protocol will necessarily be iterative.

PIESNO[1] encompasses both a *global* and an *iterative*[2] methods for identifying noise-only pixels and estimating the Gaussian noise SD from a commonly used data structure in MRI, see figure below.



[1] This work is currently in press at the Journal of Magnetic Resonance. DOI

[2] The terms *global* and *iterative* will be made precise later.

Input parameters:
N = the number of receiver coils
K = the number of images (of the same slice location)
λ_- = lower threshold value of s
λ_+ = upper threshold value of s
σ_g = initial estimate of the Gaussian noise SD
$m_{i,j,k}$ = the volumetric data set.

Output parameters:
new estimate σ_g
noise ensemble Ω
quaternary or binary mask

Step 1. Compute s_{ij} for each pixel at (i, j):

$$s_{ij} = \frac{1}{2\sigma^2 K} \sum_{k=1}^{K} m_{ijk}^2$$

A positive identification is noted at (i, j) if s_{ij} satisfies the inequalities: $\lambda_- \leq s_{ij} \leq \lambda_+$

Step 2. Construct, Ω, which is the union of all of the positively identified arrays of K measurements, and compute its sample median, $q_{1/2}$ (or optimal quantile, $q_{\alpha*}$).

Step 3. Estimate the Gaussian noise SD from the sample median (or the optimal quantile):

$$\sigma_g = \frac{q_{\alpha*}}{\sqrt{2P_s^{-1}(\alpha*|N,1)}}.$$

Step 4. Exit the iteration if Ω is empty, otherwise iterate

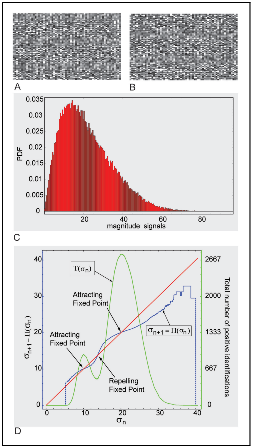
through Steps 1 to 4 and use the newly estimated σ_g in Step 1 until convergence or the maximum number of iterations is reached.

Outline

- Define two functions, $\Pi(\sigma_n)$ and $T(\sigma_n)$.
- Π takes in an estimate of σ_n and returns a new estimate of σ_{n+1}.
- Π uses only Step 1, 2 and 3. It is not iterative!
- T takes in σ_n and returns the total number of positive identifications.
- T uses only Step 1. So, it is highly efficient.

Global PIESNO
An example

- A synthetic volumetric data of $64 \times 64 \times 16$ was constructed.
- Each of the 16 images was created with random numbers drawn from two Rayleigh distributions, $\sigma = 10$ and $\sigma = 20$, based on the following rule.
- If both indices of a pixel location, (i, j), are even number, then a random number drawn from Rayleigh distribution of $\sigma = 10$ will be placed on this pixel location; otherwise, the random number will be drawn from the other Rayleigh distribution.
- Figures A and B are representative of the noisy images.
- Figure C is a histogram of the whole volumetric data. Note that it is impossible to infer from this histogram that there are two (Rayleigh) distributions!
- With *Global* PIESNO, it is possible to detect multiple Rayleigh distributions.
- In Figure D, T peaks near the attracting fixed points. The nonzero trough of T seems to coincide with the repelling fixed points.
- The results showed that the number of attracting fixed points coincides with the number of clusters of noise.

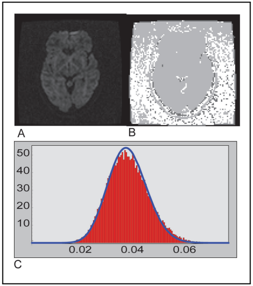


Outline

Iterative PIESNO

- Iterative PIESNO uses Step 1, 2, 3, and 4 iteratively until convergence is reached.
- Iterative PIESNO can be made automatic by a systematic search for a good initial estimate, please refer to the paper for further information.

- A set of human brain data was acquired on a 1.5 Tesla scanner (GE Medical Systems, Milwaukee, WI) based on the sum-of-squares reconstruction (8) with an 8-channel phased array coil, i.e., $N = 8$, using a single-shot spin-echo EPI sequence.
- Specific experimental parameters: Field-Of-View $= 24 \times 24cm^2$, 60 slices without gaps, slice thickness $= 2.5mm$, matrix size $= 96 \times 96$.
- 12 diffusion-weighted (DW) images were acquired with different gradient directions but with the same diffusion weighting of $1100s/mm^2$, i.e., $K = 14$ at each slice location. Figure A is a representative DW image.
- Figure B is a quaternary image. Pixels that are classified as containing noise-only measurements are shown in white. Pixels whose s_{ij}'s are greater than the upper threshold are shown in light gray. Pixels whose s_{ij}'s are nonzero but less than the lower threshold are shown in dark gray, and finally, pixels whose s_{ij}'s are zero are shown in black.
- The histogram of Ω is shown in Figure C. The probability density function of noise based on the final estimate of the Gaussian noise SD is shown as the red curve.



Iterative + Global PIESNO

Another example

- With the final estimate, σ_g, obtained from Iterative PIESNO, it may be informative to use Global PIESNO to compute the quaternary mask with different α levels (The α level changes the threshold values,λ_- and λ_+).
- The movie of the right panel shows how the quaternary image changes and α increases.

Discussion

- The initial motivation behind this work was simply to take advantage of the multiplicity of the images within the proposed data structure to increase the discriminative power of the identification of noise-only pixels, and to develop a simple and self-consistent framework of noise assessment that incorporates both the identification of noise-only pixels and the estimation of Gaussian noise SD
- The present method for estimating the Gaussian noise SD and the technique proposed in (9) represent our major attempt to decouple the fixed point formula of SNR (10) into two self-consistent approaches for estimating the underlying signal and the Gaussian noise SD.
- The present method is capable to detecting multiple noise clusters.
- The work is applicable to phased array MRI systems that use the sum-of-squares reconstruction (8). Future work may involve theoretical studies that help extend the present paradigm to other reconstruction methods. Extensive empirical work on the characterization of noise in MR images reconstructed by other methods can be found in (11).
- PIESNO may not perform when K is small, e.g., $K < 5$.

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- Software used to generate this presentation were: Beamer, LaTex, WinEdt.
- Computational and graphical software used in this work were: Mathematica, Java, Adobe Illustrator.
- PIESNO is available in Java as an independent module. This module can be called from Matlab, IDL, and Mathematica. For further information, please visit `http://sites.google.com/site/piesnoformri`.
- PIESNO is included in `http://science.nichd.nih.gov/confluence/display/nihpd/TORTOISE`.